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Return to Equilibrium and Stability of Dynamics (Semigroup Dynamics Case)

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A study is presented of asymptotically normal dynamical semigroups for which there exists a faithful normal state satisfying the detailed balance condition. Such dynamics reveals a return to a stationary state if additionally a weak cluster property is assumed. The generalized stability condition and generalized wave operators are introduced. The theory is illustrated by models.

KEY WORDS: Return to equilibrium; dynamical semigroup; algebra of observables; detailed balance condition; KMS condition.

1. INTRODUCTION

Recently, the generalization of scattering theory ideas to the framework of C^*-algebra has made very interesting progress.[1–3] These investigations assume reversible dynamics, i.e., that the dynamics under consideration is given by a one-parameter group of automorphisms on a C^*-algebra.

On the other hand, analysis of scattering theory for systems with irreversible dynamics has been carried out, i.e., dynamics is assumed to be given by a one-parameter semigroup of linear transformations, and the above-mentioned results are formulated in the Hilbert space language.[4–6] For that reason it is interesting to ask whether one can combine the C^*-algebraic approach in the scattering theory with the assumption of irreversibility of dynamics. My aim is to give an affirmative answer to this question. To be more specific, I will discuss the relation of scattering theory to the approach to equilibrium in statistical mechanics. This will be done under the additional assumptions that the considered dynamics is asymptotically normal as well as that there exists a faithful state which

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satisfies the detailed balance condition with respect to the semigroup dynamics τ_t. Furthermore, the study of "generalized" wave operators and results concerning the stability of dynamics are presented. In the final section I present comments and illustrative models. I want to point out that one can consider part of these results as an extension of Tropper's results[7] to a quantum model.

For a detailed description of the dynamical semigroup in statistical mechanics and physical models see Majewski.[8]

2. PRELIMINARIES AND ASSUMPTIONS

Let $\mathfrak{M}$ be a W^*-algebra. We assume that $\mathfrak{M}$ acts on a complex Hilbert space $\mathscr{H}$ and that $\Omega \in \mathscr{H}$ is a cyclic and separating vector for $\mathfrak{M}$. The faithful state on $\mathfrak{M}$ defined by Ω will be denoted by ω, i.e., $\omega(A) = (\Omega, A\Omega)$ for $A \in \mathfrak{M}$. Let τ_t be a strongly positive[2] $[\tau_t(A^*)\,\tau_t(A) \leqslant \tau_t(A^*A)$ for $A \in \mathfrak{M}]$ dynamical semigroup on $\mathfrak{M}$. In particular, τ_t is a normal map for each $t > 0$. The triple $(\mathfrak{M}, \tau_t, \omega)$ will be called a dynamical system when ω is the τ-invariant (normal, faithful) state.

We restrict ourselves to the class of dynamical systems satisfying the following two conditions:

Condition I (Detailed balance condition[9]). A normal faithful state ω of $\mathfrak{M}$ satisfies the detailed balance condition with respect to a dynamical semigroup τ_t whenever

$$\omega \circ \tau_t = \omega, \qquad t \geqslant 0 \tag{1}$$

$$\omega(A^*\tau_t(B)) = \omega(\sigma(B^*)\,\tau_t\sigma(A)), \qquad A, B \in \mathfrak{M}; \quad t \geqslant 0 \tag{2}$$

where σ denotes a reversing operation on $\mathfrak{M}$, i.e., $\sigma\colon \mathfrak{M} \to \mathfrak{M}$ is an antilinear Jordan automorphism of order two.

Condition II. A dynamical system is called asymptotically normal if

$$\lim_{t \to \infty} \omega(\tau_t(A)\,\tau_t(B)) = \lim_{t \to \infty} \omega(\tau_t^\sigma(A)\,\tau_t^\sigma(B)) \tag{3}$$

for $A, B \in \mathfrak{M}$, where $\tau_t^\sigma = \sigma \circ \tau_t \circ \sigma$ and σ is the reversing operation.

In the Appendix we give a model which cleary shows that detailed balance condition and Condition II are compatible.

[2] Note that as τ_t is positive, it is automatically self-adjoint, i.e., $\tau_t(A^*) = \tau_t(A)^*$ for all $A \in \mathfrak{M}$. See, e.g., Kadison's argument in Lemma 8, ref. 22.

The definition

$$\hat{\tau}_t A\Omega \stackrel{\text{df}}{=} \tau_t(A)\,\Omega$$

gives a weakly continuous semigroup of contractions on $\mathscr{H}$ such that $\hat{\tau}_t\Omega = \Omega$, $t \geqslant 0$. In the sequal, $\hat{\tau}_t$ will be called a dynamical semigroup on the Hilbert space $\mathscr{H}$.

As Ω is a cyclic and separating vector for $\mathfrak{M}$, there exists a modular operator Δ and a modular conjugation $\mathscr{J}_\Omega$ by the Tomita–Takesaki theory. Let V_α be the weak closure of the set of vectors $\{\Delta^\alpha A\Omega; A \in \mathfrak{M}, A \geqslant 0\}$, where $\alpha \in [0, 1/2]$. The quantity $V_{1/4}$ will be denoted by $\mathscr{P}$ and called the natural cone.

An important property of V_α is that the dual of V_α is $V_{1/2-\alpha}$ ($\alpha \in [0, 1/4]$).

In the sequal, the set of all states on $\mathfrak{M}$ will be denoted by $S(\mathfrak{M})$, and the set of all normal states on $\mathfrak{M}$ will be denoted by $\mathscr{S}(\mathfrak{M})$. We will need the following subset of $\mathscr{S}(\mathfrak{M})$:

$$\mathfrak{M}_*^0 = \{\varphi \in \mathscr{S}(\mathfrak{M});\ \varphi \leqslant \alpha\omega \text{ for a positive number } \alpha\}$$

It is easy to see that $\mathfrak{M}_*^0$ is a norm dense subset of $\mathscr{S}(\mathfrak{M})$. The linear version of the Radon–Nikodym theorem states that there exists a $\phi_{(\varphi)} \in V_0$ such that $\alpha\Omega - \phi_{(\varphi)} \in V_0$ and $2\varphi(A) = (\phi_{(\varphi)} A\Omega) + (\Omega, A\phi_{(\varphi)})$ for any state $\varphi \in \mathfrak{M}_*^0$ and $A \in \mathfrak{M}$.

Finally, we will need also the conjugation $\mathscr{J}: \mathscr{H} \to \mathscr{H}$ induced by σ, i.e., $\mathscr{J} A\Omega = \sigma(A)\,\Omega$ for $A \in \mathfrak{M}$. (in general, $\mathscr{J}_\Omega \neq \mathscr{J}$).

3. RETURN TO EQUILIBRIUM

In this section we examine the long-time behavior of the dynamical system $(\mathfrak{M}, \tau_t, \omega)$ for which Conditions I and II are satisfied.

As a starting point, let us consider the question of existence of the weak*-limit of $\tau_t^*\varphi$, where φ is a normal state and τ_t^* denotes the dual semigroup, i.e., $(\tau_t^*\varphi)(A) = \varphi(\tau_t(A))$ for $A \in \mathfrak{M}$, $t \geqslant 0$, $\varphi \in \mathfrak{M}_*$.

We will need the following subset of $\mathfrak{M}$:

$$\mathfrak{N}(\tau) = N(\tau) \cap N(\tau)^+$$

where

$$N(\tau) = \{A \in \mathfrak{M};\ \tau_t(A^*A) = \tau_t(A)^*\,\tau_t(A),\ t > 0\}$$

$$N(\tau)^+ = \{A \in \mathfrak{M};\ \tau_t(AA^*) = \tau_t(A)\,\tau_t(A)^*,\ t > 0\}$$

Note that $\mathfrak{N}(\tau)$ is the largest τ-invariant W^*-subalgebra of $\mathfrak{M}$ on which τ_t is equal to a group of automorphisms.[10]

Remark. To prove that τ_t maps $\mathfrak{N}(\tau)$ *onto* $\mathfrak{N}(\tau)$ it is enough to examine τ_t as a normal, faithful *-morphism $\tau_t: \mathfrak{N}(\tau) \to \mathfrak{N}(\tau)$ such that

$$(\Omega, A\Omega) = (\Omega, \tau_t(A)\,\Omega)$$

for $A \in \mathfrak{N}(\tau)$, where $\Omega \in \overline{\mathfrak{N}(\tau)\,\Omega} = \mathscr{H}_1$ is a cyclic and separating vector for $\mathfrak{N}(\tau)$.

Further, let us reformulate Conditions I and II (see Section 2). Condition I implies

$$\hat{\tau}_t \mathfrak{M}^+ \Omega \subset \mathfrak{M}^+ \Omega \qquad \text{and} \qquad \mathscr{J} V_0 \subset V_0$$

where $\mathfrak{M}^+ = \{A \in \mathfrak{M}; A \geqslant 0\}$. Moreover (see p. 40 in ref. 11)

$$s\text{-}\lim_{t \to +\infty} \hat{\tau}_t^* \hat{\tau}_t = Q^\tau$$

always exists.

It can be noticed that $\hat{\tau}_t^* \hat{\tau}_t \mathfrak{M}^+ \Omega \subset \mathfrak{M}^+ \Omega$. Hence $Q^\tau \mathfrak{M}^+ \Omega \subset \overline{\mathfrak{M}^+ \Omega}$ and then[12] $Q^\tau \mathfrak{M}^+ \Omega \subset \mathfrak{M}^+ \Omega$. Thus, by another result of Bratteli and Robinson[12] there exists a positive map q^τ of $\mathfrak{M}$ into $\mathfrak{M}$ such that

$$q^\tau(A)\,\Omega = Q^\tau A \Omega$$

for all $A \in \mathfrak{M}$. On the other hand,

$$\lim_{t \to \infty} \omega(\tau_t(A)\,\tau_t(B)) = \omega(q^\tau(A)\,B))$$

$$\lim_{t \to \infty} \omega(\tau_t^\sigma(A)\,\tau_t^\sigma(B)) = \omega(\sigma \circ q^\tau \circ \sigma(A)\,B)$$

Therefore, for the dynamical system $(\mathfrak{M}, \tau_t, \omega)$ with τ_t-invariant, faithful, normal state ω satisfying the detailed balance condition the Condition II is equivalent to the following property of σ:

$$\sigma \cdot q^\tau \cdot \sigma = q^\tau \tag{$*$}$$

Remarks. (i) It should be noted that the set of all reversing operators σ is now restricted by condition $(*)$. This can be considered as a clarification of the meaning of Condition II.

(ii) In the Appendix we give an example which clearly shows that the detailed balance condition and condition $(*)$ do not exclude each other, but are easily compatible.

Now we can formulate the following result.

Theorem. Let $(\mathfrak{M}, \tau_t, \omega)$ be a dynamical system. Furthermore, let us assume that ω satisfies the following:

(i) The detailed balance condition with respect to τ and such a fixed reversing operation σ that

$$\sigma q^{\tau} \sigma = q^{\tau}$$

(ii) There is a limit of the function

$$\mathbb{R}^{+} \ni t \to \omega(A\tau_t(B))$$

when $t \to \infty$ for all $B \in \mathfrak{N}(\tau)$ and all $A \in \mathfrak{M}_0$ (where $\mathfrak{M}_0$ is a σ-weakly dense τ-invariant *-subalgebra of $\mathfrak{M}$). Then, it follows that the limit

$$\varphi_{+}(A) = \lim_{t \to +\infty} \varphi(\tau_t(A))$$

exists for all $A \in \mathfrak{M}$ and all normal states on $\mathfrak{M}$, i.e., the system $(\mathfrak{M}, \tau_t, \omega, \varphi)$ manifests "a return to equilibrium."

Remarks. (i) Let us observe that the condition (i) of the theorem is satisfied for the semigroup evolution τ_t described in our model (see Section 4); $\hat{\tau}_t$ in this model is in fact a self-adjoint semigroup.

(ii) One can say that condition $(*)$ or equivalently Condition II means the asymptotic normality of $\hat{\tau}_t$, i.e., $s\text{-}\lim_{t\to\infty} \hat{\tau}_t^* \hat{\tau}_t = s\text{-}\lim_{t\to\infty} \hat{\tau}_t \hat{\tau}_t^*$ (see the proof of the lemma below).

We will need the following lemma.

Lemma. Adopt the assumptions of the theorem. Then

$$w\text{-}\lim_{t \to +\infty} \hat{\tau}_t^* \hat{\tau}_t = Q^{\tau}(\equiv Q)$$

is an orthogonal projection on a subspace of $\mathscr{H}$.

Moreover $\lim_{t\to\infty} \|\hat{\tau}_t(\mathbb{1} - Q) f\| = 0$ for any $f \in \mathscr{H}$ and $\hat{\tau}_t|_{Q\mathscr{H}}$ is a one-parameter unitary semigroup on $Q\mathscr{H}$.

Proof. First, let us observe that

$$\begin{aligned}
\omega(\sigma q^{\tau} \sigma(A)\, B) &= \omega(\sigma(B^*)\, q^{\tau} \sigma(A^*)) \\
&= \lim_{t \to +\infty} (\mathscr{J} B\Omega, \hat{\tau}_t^* \hat{\tau}_t \mathscr{J} A^* \Omega) \\
&= \lim_{t \to +\infty} (\hat{\tau}_t \mathscr{J} B\Omega, \hat{\tau}_t \mathscr{J} A^* \Omega) \\
&= \lim_{t \to +\infty} (\mathscr{J} \hat{\tau}_t \mathscr{J} A^* \Omega, \mathscr{J} \hat{\tau}_t \mathscr{J} B\Omega) \\
&= \lim_{t \to +\infty} (\hat{\tau}_t \hat{\tau}_t^* A^* \Omega, B\Omega)
\end{aligned}$$

for $A, B \in \mathfrak{M}$. Hence, it is easy to see that condition $(*)$ implies

$$\lim_{t \to +\infty} (\hat{\tau}_t \hat{\tau}_t^* A^* \Omega, B\Omega) = \omega(\sigma q^\tau \sigma(A) B) = \omega(q^\tau(A) B)$$

$$= \lim_{t \to +\infty} (\hat{\tau}_t^* \hat{\tau}_t A^* \Omega, B\Omega) \tag{4}$$

Next recall that for any contraction semigroup τ_t on $\mathscr{H}$, s-lim $\hat{\tau}_t^* \hat{\tau}_t = Q$ exists and obviously $Q \geqslant 0$, $\|Q\| \leqslant 1$. The equalities (4) imply

$$s\text{-}\lim_{t \to +\infty} \hat{\tau}_t^* \hat{\tau}_t = s\text{-}\lim_{t \to +\infty} \hat{\tau}_t \hat{\tau}_t^* = Q \tag{5}$$

Further, let us observe that

$$\hat{\tau}_s Q \hat{\tau}_s^* = \lim_{t \to +\infty} \hat{\tau}_s \hat{\tau}_t \hat{\tau}_t^* \hat{\tau}_s^* = \lim_{t \to +\infty} \hat{\tau}_{t+s} \hat{\tau}_{t+s}^* = Q$$

for any $s > 0$. Also, for any $s > 0$,

$$\hat{\tau}_s^* Q \hat{\tau}_s = \lim_{t \to +\infty} \hat{\tau}_s^* \hat{\tau}_t^* \hat{\tau}_t \hat{\tau}_s = \lim_{t \to +\infty} \hat{\tau}_{t+s}^* \hat{\tau}_{t+s} = Q$$

Therefore

$$\hat{\tau}_s Q \hat{\tau}_s^* = Q$$

$$\hat{\tau}_s^* \hat{\tau}_s Q \hat{\tau}_s^* = \hat{\tau}_s^* Q$$

$$\hat{\tau}_s^* \hat{\tau}_s Q \hat{\tau}_s^* \hat{\tau}_s = \hat{\tau}_s^* Q \hat{\tau}_s = Q$$

Hence

$$Q = w\text{-}\lim_{s \to +\infty} (\hat{\tau}_s^* \hat{\tau}_s Q^{1/2})(Q^{1/2} \hat{\tau}_s^* \hat{\tau}_s) = Q^3$$

Thus, using the spectral theorem, it is easy to see that

$$Q = \int_{\sigma(Q)} \lambda \, dE(\lambda)$$

with $\sigma(Q) = \{0, 1\}$. But this means that Q is an orthogonal projector.

Now, let us consider

$$\lim_{t \to +\infty} \|\hat{\tau}_t(\mathbb{1} - Q) f\|^2 = \lim_{t \to +\infty} (\hat{\tau}_t f - \hat{\tau}_t Q f, \hat{\tau}_t f - \hat{\tau}_t Q f)$$

$$= (Qf, f) - (Qf, Qf) - (Q^2 f, f) + (Qf, Q^2 f) = 0$$

Next, one finds that

$$\lim_{t\to+\infty}\|\hat{\tau}_t Qf\|^2=\lim_{t\to+\infty}(Qf,\hat{\tau}_t^*\hat{\tau}_t Qf)=\|Qf\|^2$$

Since $\hat{\tau}_t$ is the semigroup of contractions, then the above equalities imply

$$\|\hat{\tau}_t Qf\|=\|Qf\|$$

for any $t>0$. Furthermore, condition $(*)$ implies

$$\mathscr{J}Q\mathscr{J}=Q$$

Hence, for $t>0$,

$$\begin{aligned}\|\hat{\tau}_t^*Qf\|&=\|\mathscr{J}\hat{\tau}_t\mathscr{J}Qf\|=\|\hat{\tau}_tQ\mathscr{J}f\|\\&=\|Q\mathscr{J}f\|=\|\mathscr{J}Qf\|=\|Qf\|\end{aligned}$$

Therefore

$$Q\mathscr{H}=\{f\in\mathscr{H};\ \|\hat{\tau}_tf\|=\|f\|,\ t\geqslant 0\}\cap\{f\in\mathscr{H};\ \|\hat{\tau}_t^*f\|=\|f\|,\ t\geqslant 0\}$$

Applying the argument given in ref. 11, p. 9, one concludes that $Q\mathscr{H}$ is $\hat{\tau}$-invariant and $\hat{\tau}|_{Q\mathscr{H}}$ is a one-parameter unitary group on $Q\mathscr{H}$.

Proof of Theorem. Let φ be in $\mathfrak{M}_*^0$ (cf. Section 2). Therefore

$$\begin{aligned}2\varphi(\tau_t(A))&=(\phi,\tau_t(A)\,\Omega)+(\Omega,\tau_t(A)\,\phi)\\&=(\phi,\hat{\tau}_tA\Omega)+(\hat{\tau}_tA^*\Omega,\phi)\end{aligned}$$

where $\phi\in V_0$. The lemma implies

$$\lim_{t\to+\infty}(\tau_t^*\varphi)(A)=\tfrac{1}{2}\lim_{t\to+\infty}[(\phi,\hat{\tau}_tQA\Omega)+(\hat{\tau}_tQA^*\Omega,\phi)]$$

Observe that for any $B\in\mathfrak{N}(\tau)$, $A\in\mathfrak{M}$,

$$(\hat{\tau}_t^*\hat{\tau}_tB\Omega,A\Omega)=(\hat{\tau}_tB\Omega,\hat{\tau}_tA\Omega)=(B\Omega,A\Omega)$$

where the second equality follows from the automorphism property of $\tau|_{\mathfrak{N}(\tau)}$ and the Cauchy–Schwarz inequality applied to the forms

$$f_t(A,B)=(B\Omega,A\Omega)-(\hat{\tau}_tB\Omega,\hat{\tau}_tA\Omega)$$

Therefore

$$((\mathbb{1}-Q)\,B\Omega,A\Omega)=\lim_{t\to+\infty}((\mathbb{1}-\hat{\tau}_t^*\hat{\tau}_t)\,B\Omega,A\Omega)=0$$

Hence

$$Q\mathfrak{N}(\tau)\,\Omega = \mathfrak{N}(\tau)\,\Omega \tag{6}$$

Next, let us recall that one can define a positive map q of $\mathfrak{M}$ into $\mathfrak{M}$ such that $q(A)\,\Omega = QA\Omega$ for all $A \in \mathfrak{M}$. From ref. 10, p. 138, it follows that q maps $\mathfrak{M}$ onto $\mathfrak{N}(\tau)$ and $Q \leqslant [\mathfrak{N}(\tau)\,\Omega]$. Thus, this result and formula (6) imply

$$Q = \text{projector on the closure of } \mathfrak{N}(\tau)\,\Omega \tag{7}$$

Consequently, Condition (ii) of the theorem and formula (7) imply $\lim_{t \to +\infty}(\tau_t^* \varphi)(A)$ exists for $\varphi \in \mathfrak{M}_*^0$ and $A \in \mathfrak{M}$. Thus, the statement of the theorem follows immediately, since the subset $\mathfrak{M}_*^0$ is dense in the set of all normal states of $\mathfrak{M}$.

Remark. Condition (ii) of the theorem is a weak cluster property of the state. There are stronger cluster properties (see, for example, Robinson[10]) which are equivalent to τ-ergodicity of ω. It seems fair, therefore, to say that Condition (ii) means that ω should have a relatively "pure" form. Moreover, note that Condition (ii) is a weak cluster property since there is the restriction to subalgebra $\mathfrak{N}(\tau)$, i.e., to the largest subset of observables on which the semigroup time evolution is, in fact, the reversible one.

The above theorem leads directly to the following corollary.

Corollary. Let the dynamical system $(\mathfrak{M}, \tau_t, \omega)$ satisfy Condition (i) of the theorem. Moreover, let

$$\lim_{t \to +\infty} \omega(A\tau_t(B)) = \omega(A)\,\omega(B) \tag{8}$$

for all $A \in \mathfrak{M}_0$, $B \in \mathfrak{N}(\tau)$, where the above limits exist uniformly in $B \in \mathfrak{N}(\tau)$.

Then, the following limits exist:

1. $w\text{-}\lim_{t \to +\infty} \hat{\tau}_t \Delta^{-it}$

2. $\lim_{t \to +\infty} \varphi(\tau_t \circ \sigma_{-t}(A))$ for $\varphi \in \mathscr{S}(\mathfrak{M})$ and $A \in \mathfrak{M}$. Here $\sigma_t(\cdot)$ denotes the group of modular automorphisms on $\mathfrak{M}$, i.e., $\sigma_t(A) = \Delta^{it} A \Delta^{-it}$, $t \in \mathbb{R}$.

Proof. By the equality (8),

$$\lim_{t \to +\infty} \omega(A\tau_t(\sigma_{-t}(B))) = \omega(A)\,\omega(B)$$

for $A \in \mathfrak{M}_0$, $B \in \mathfrak{N}(\tau)$. Hence

$$\lim_{t \to +\infty} (\psi, \hat{\tau}_t \Delta^{-it} B\Omega)$$

exists for $\psi \in \{A\Omega; A \in \mathfrak{M}_0\}$, $B \in \mathfrak{N}(\tau)$. Consequently, $w\text{-}\lim_{t\to+\infty} \hat{\tau}_t \Delta^{-it}$ exists [since $\hat{\tau}_t(\mathbb{1} - Q)\mathscr{H} \to^s 0$]. The second statement follows directly from the first one and the Radon–Nikodym theorem described in Section 2.

Remark. Let us consider the strong mixing property

$$\lim_{t\to+\infty} \omega(A\tau_t(B)) = \omega(A)\,\omega(B), \qquad A, B \in \mathfrak{M} \tag{9}$$

Clearly, we have used this kind of assumption in our corollary, and (9) is a much stronger cluster property of the dynamical system $(\mathfrak{M}, \tau_t, \omega)$ than we have assumed in the theorem. Thus, it is worth mentioning an interplay between strong mixing and weak asymptotic Abelianess.

Namely, if in addition to the assumed properties of the dynamical system $(\mathfrak{M}, \tau_t, \omega)$ we would add that ω is a primary state, then a slight modification of argument given in ref. 2, p. 396 shows that weak asymptotic Abelianess,

$$\omega(C[A, \tau_t(B)]\,D) \xrightarrow[t\to+\infty]{} 0$$

A, B, C, $D \in \mathfrak{M}$, and strong mixing are actually equivalent (see also Winnink[13]).

Now, we can formulate the following.

Definition. We define the map $\gamma_+^* : \mathscr{S}(\mathfrak{M}) \to S(\mathfrak{M})$ by

$$(\gamma_+^* \varphi)(A) = \phi(A) = \lim_{t\to+\infty} ((\tau_t \circ \sigma_{-t})^* \varphi)(A)$$

for $A \in \mathfrak{M}$, and the map $\gamma_+ : \mathfrak{M} \to \mathfrak{M}$ by transposition, i.e.,

$$\varphi(\gamma_+(A)) = \lim_{t\to+\infty} \varphi(\tau_t \circ \sigma_{-t}(A))$$

for $A \in \mathfrak{M}$, $\varphi \in \mathscr{S}(\mathfrak{M})$.

Now let us comment on the above definition. The scattering theory normally involves a comparison of two different dynamics of the same system: the given dynamics describing an interacting system and a "free" dynamics. We assume that the given dynamics is a dynamical semigroup. Usually such semigroups arise when the uncontrollable influence from the outside is taken into account in a phenomenological manner by including absorptive or decay terms in the interaction. On the other hand, the Tomita–Takesaki theory implies that the state ω satisfies the KMS condition with respect to the modular dynamics. Thus, one would believe that the modular dynamics is a proper candidate for the "free" dynamics. *Therefore, the map* γ_+ introduced by the above definition *is the algebraic* *analogue of the wave operators* of scattering theory. (Observe that the corollary implies the correct definition of γ_+.), It is straightforward to check that γ_+ is a linear unital and positive map, but γ_+ fails in general to be a normal transformation. The physical motivation for the last result is that, for example, the infared phenomenon is expected to destroy normality. Finally, let us note that γ_+ satisfies the interwining relation:

$$\tau_t \gamma_+ = \gamma_+ \sigma_t$$

Next, we study when and in what sense $\hat{\tau}_t$ is a perturbation of the modular dynamics. The following considerations are elementary, but essential in what follows. The detailed balance condition implies[9] (see Section 2)

$$\Delta^{-it} \hat{\tau}_s = \hat{\tau}_s \Delta^{-it}$$

for $t, s \geqslant 0$. Let $f \in D(\ln \Delta)$, where $i \ln \Delta$ is the infinitesimal generator of the modular group Δ^{it}. Then, one has

$$\frac{d}{dt} \hat{\tau}_s \Delta^{-it} f = \frac{d}{dt} \Delta^{-it} \hat{\tau}_s f$$

and both terms in the above equality exist. Thus,

$$\hat{\tau}_s D(\ln \Delta) \subset D(\ln \Delta)$$

$$\hat{\tau}_s \ln \Delta \, f = \ln \Delta \, \hat{\tau}_s f$$

for $f \in D(\ln \Delta)$. Furthermore, the semigroup theory implies

$$(R_\lambda(S) \ln \Delta \, f, g) = (R_\lambda(S) f, \ln \Delta \, g) \tag{10}$$

where $f, g \in D(\ln \Delta)$, S is the infinitesimal generator of $\hat{\tau}_t$, λ is a complex number such that $\operatorname{Re} \lambda > 0$, and finally $R_\lambda(S)$ denotes the resolvent of S at λ.

Let us denote $\{R_\lambda(S)\, D(\ln \Delta)\}$ by $\mathscr{D}$. We deduce directly from (10)

$$\mathscr{D} \subset \mathscr{D}(\ln \Delta) \cap \mathscr{D}(S)$$

and

$$R_\lambda(S) \ln \Delta \, f = \ln \Delta \, R_\lambda(S) f$$

for $f \in D(\ln \Delta)$.[12] We have thus shown that there is a dense subset $\mathscr{D} \subset D(S) \cap D(\ln \Delta)$ invariant with respect to $\hat{\tau}_t$ and Δ^{-it}. Therefore, the set $\mathscr{D}$ is a core for the infinitesimal generator of the semigroup $\hat{\tau}_t \Delta^{-it}$ (cf. Corollary 3.1.7 in ref. 2).

Hence, the definition

$$S - i \ln \Delta |_{\mathscr{D}} \tag{11}$$

makes sense and describes the restriction of the infinitesimal generator to the core $\mathscr{D}$. Moreover, we can conclude that: *The expression (11) describes the perturbation of the modular group Δ^{-it} and this perturbation is such that the perturbed modular dynamics is equal to the dynamical semigroup $\hat{\tau}_t$.*

Next, let us consider σ_t-invariant state φ which belongs to $\mathfrak{M}_*^0$,

$$2\varphi(\tau_t(A)) = (\phi, \Delta^{-it}\hat{\tau}_t A\Omega) + (\Delta^{-it}\hat{\tau}_t A^*\Omega, \phi)$$

where $\phi \in V_0$. Assume $A^*\Omega$ and $A\Omega \in \mathscr{D}$ (see the above discussion for definition of $\mathscr{D}$). Let us observe

$$\begin{aligned} &2 \lim_{t \to +\infty} \varphi(\tau_t(A)) - 2\varphi(A) \\ &\quad = \int_0^\infty dt(\phi, \Delta^{-it}(S - i \ln \Delta)\, \hat{\tau}_t A\Omega) \\ &\quad\quad + \int_0^\infty dt(\Delta^{-it}(S - i \ln \Delta)\, \hat{\tau}_t A^*\Omega, \phi) \end{aligned}$$

Thus, the detailed balance condition and the cluster property of ω imply the integrability condition of the following function:

$$\mathbb{R}^+ \ni t \mapsto (\phi, \Delta^{-it}(S - i \ln \Delta)\, \hat{\tau}_t A\Omega) \tag{12}$$

This condition is of interest because it *can be considered as a generalization of the celebrated stability condition*, which was introduced by Robinson[1] for the reversible dynamics. Hence, one can conclude that the detailed balance condition imposes a stability condition on the behavior of the pair (τ_t, ω).

We end this section with some concluding remarks. First of all, we wish to emphasize that the above-discussed stability property of the pair (ω, τ_t) follows as a result of the assumed detailed balance condition for (ω, τ_t). In other words, this result is not a property of the perturbed modular system associated with the pair $(\mathfrak{M}, \Omega)$. To see this fact, it is enough to observe that we have assumed a very weak form of a cluster property for (ω, τ_t) and the assumed cluster condition involves the subalgebra $\mathfrak{N}(\tau)$. Moreover, in our proofs, Conditions I and II have played an important role (compare the theorem and the corollary). It should be noted that according to Davies (ref. 14, §6.4) and Sz-Nagy and Foiaş (ref. 11, p. 274), it is not the general case that $\hat{\tau}_t^* \hat{\tau}_t$ converges strongly to a projection as $t \to +\infty$.

On the other hand, for a weakly asymptotically Abelian system (see remark following the corollary) with a factorial faithful state it is possible to find such perturbations of modular dynamics that the generalized wave operators exist. But such a point of view demands a *stronger ergodic* assumption about the physical system than we have made.

4. MODEL. QUANTUM HARMONIC OSCILLATOR

Let $\mathcal{A}$ be a C^*-algebra generated by nonzero elements $W(z)$, $z \in \mathbb{C}$, satisfying

$$W(z)^* = W(-z)$$

$$W(z)\, W(z') = \{\exp[-i \operatorname{im}(z \cdot \bar{z}')/2]\}\, W(z+z')$$

for all z, $z' \in \mathbb{C}$. $\mathcal{A}$ is the CCR algebra over one-dimensional Hilbert space. Let us consider the map

$$z \mapsto z_t = e^{-i\omega_0 t} z$$

for $t \in \mathbb{R}$, $\omega_0 > 0$, $z \in \mathbb{C}$. Obviously, there exists a unique one-parameter group of automorphisms α_t^0 of $\mathcal{A}$ such that

$$\alpha_t^0(W(z)) = W(z_t)$$

Further, let us define the following state over $\mathcal{A}$:

$$\psi_\beta(W(z)) = \exp(-Q_\beta\, |z|^2/4)$$

where $Q_\beta = (1+e^{-\beta\omega_0})(1-e^{-\beta\omega_0})^{-1}$ for $\beta > 0$.

Let us take the GNS triple $(\mathcal{H}, \pi(\mathcal{A}), \Omega_\beta)$ associated with $(\mathcal{A}, \psi_\beta)$ and subsequently take the weak closure of $\pi(\mathcal{A})$ on $\mathcal{H}$. Thus, we obtain the concrete von Neumann algebra $\mathfrak{M}$ on $\mathcal{H}$. Clearly, $\psi_\beta \cdot \alpha_t^0 = \psi_\beta$. Hence, α_t^0 induces a unitary implemented one-parameter group of automorphisms of $\pi(\mathcal{A})$ which can be extended over $\mathfrak{M}$. We will denote the extension by α_t. It is easy to verify that the state $\tilde{\psi}_\beta(A) = (\Omega_\beta, A\Omega_\beta)$, $A \in \mathfrak{M}$, satisfies the KMS condition with respect to α_t. Thus, one can treat the α_t evolution and the modular evolution as identical.[15,16,23]

Further, the reversing operation σ is defined in the following way:

$$\tilde{\sigma}(W(z)) = W(\bar{z}), \qquad z \in \mathbb{C}$$

and consequently $\sigma = \pi \circ \tilde{\sigma} \circ \pi^{-1}$. Now, we introduce a semigroup time evolution τ_t, which one can interpret as describing the diffusion of a quantum particle in a harmonic well,[17]

$$\tau_t\colon \quad W_\pi(z) \mapsto W_\pi([\exp(-\lambda t)]\, z) \exp\{-\tfrac{1}{4} Q_\beta\, |z|^2\, [1-\exp(-2\lambda t)]\}$$

where λ is a positive fixed constant, $t > 0$, $W_\pi(\cdot) = \pi \circ W(\cdot)$.

First note that as $\mathbb{C} \ni z \mapsto e^{-\lambda t} z \in \mathbb{C}$ is a contraction on $\mathbb{C}$, τ_t is a completely positive map, $\tau_t: \pi(\mathscr{A}) \to \pi(\mathscr{A})$. Further, for each contraction $T: \mathbb{C} \to \mathbb{C}$ there exists a contraction $F_Q(T): \mathscr{H} \to \mathscr{H}$ such that

$$F_Q(T)\, W_\pi(z)\, \Omega_\beta = \{\exp[-(Q_\beta/4)(|z|^2 - |Tz|^2)]\}\, W_\pi(Tz)\, \Omega_\beta$$

for all $z \in \mathbb{C}$. Second, let us define

$$(u_t f)(x) = e^{-ixt} f(x)$$

and choose

$$f_0(x) = \left(\frac{1}{\pi} \frac{\lambda}{\lambda^2 + x^2}\right)^{1/2} \in \mathscr{L}^2(\mathbb{R}, dx)$$

Then u_t satisfies $(f_0, u_t f_0) = e^{-\lambda t}$ and one can prove that

$$\tau_t W_\pi(z) = F_Q(V_t)^* \, W_\pi(z f_0)\, F_Q(V_t)$$

where $V_t = u_{-t} \circ i$, $i: \mathbb{C} \ni z \mapsto z f_0 \in \mathscr{L}^2(\mathbb{R}, dx)$.

The above observations are taken from Chapter 10 of ref. 18 and from ref. 17. It clearly follows that τ_t has an ultraweak extension to a completely positive map on $\mathfrak{M}$. We will denote the extension by the same letter τ. Hence, τ_t is a one-parameter strongly positive semigroup over $\mathfrak{M}$ and describes an irreversible process.[17,19] Moreover, $(\tau_t, \tilde{\psi})$ satisfies the detailed balance condition.[20]

Let φ be a normal state on $\mathfrak{M}$; then

$$\varphi(\tau_t(W_\pi(z))) = \exp\{-\tfrac{1}{4} Q_\beta \,|z|^2\, [1 - \exp(-2\lambda t)]\}\, \varphi(W_\pi([\exp(-\lambda t)]\, z))$$

Hence

$$\lim_{t \to \infty} (\tau^* \varphi)(W_\pi(z)) = \tilde{\psi}_\beta(W_\pi(z))$$

This means that an arbitrary normal state on $\mathfrak{M}$ evolves under the semigroup evolution τ_t^* to the equilibrium state $\tilde{\psi}_\beta$. Further, note

$$\lim_{t \to \infty} \tilde{\psi}_\beta(W_\pi(z)\, \tau_t(W_\pi(z'))) = \tilde{\psi}_\beta(W_\pi(z))\, \tilde{\psi}_\beta(W_\pi(z'))$$

for $z, z' \in \mathbb{C}$, so the all assumptions of the corollary are satisfied. Therefore $\lim_{t \to \infty} \varphi((\tau_t \circ \sigma_{-t})(A))$ exists. In the framework of our example one can compute this limit explicitly, and

$$\lim_{t \to \infty} \varphi((\tau_t \circ \sigma_{-t})(W_\pi(z))) = \tilde{\psi}_\beta(W_\pi(z))$$

for $z \in \mathbb{C}$.

APPENDIX

Here, we present a nontrivial model of a dynamical system for which Conditions I and II are satisfied. The model below is of a rather mathematical nature and the theory of Hilbert algebras is extensively used (see Chapter I, §6, in ref. 21).

Let $\mathscr{H}$ be a separable Hilbert space, $\mathscr{L}(\mathscr{H})$ the W^*-algebra of all linear, bounded operators on $\mathscr{H}$, and ρ be a strict positive density matrix on $\mathscr{H}$, i.e., $\rho = \sum_i \lambda_i P_{\langle x_i \rangle}$ where $P_{\langle x_i \rangle}$ are orthogonal projectors on the subspace generated by the vectors x_i, $\{x_i\}_{i \in \mathscr{N}}$ is a basis in $\mathscr{H}$, $\lambda_i > 0$ for each i, $\sum_i \lambda_i = 1$. Further, let $\mathscr{A}$ denote the set of all Hilbert–Schmidt operators on $\mathscr{H}$. Clearly, $\rho^{1/2} \in \mathscr{A}$ and $\mathscr{A}$ is a Hilbert space with respect to the following inner product:

$$((\rho, u)) = \operatorname{Tr} \rho^* u$$

for ρ, $u \in \mathscr{A}$. Now we define the following representation Π_L of $\mathscr{L}(\mathscr{H})$ in $\mathscr{L}(\mathscr{A})$:

$$\Pi_L(A)\,\sigma = A\sigma$$

for $A \in \mathscr{L}(\mathscr{H})$, $\sigma \in \mathscr{A}$. Let us denote the von Neumann algebra $\{\Pi_L(A); A \in \mathscr{L}(\mathscr{H})\}$ by $\mathfrak{M}$. It is easy to check that:

(1) $\rho^{1/2}$ is a cyclic and separating vector in $\mathscr{A}$ for $\mathfrak{M}$.

(2) The representation $(\Pi_L(\cdot), \mathscr{A}, \rho^{1/2})$ is unitary equivalent to the cyclic representation of $\mathscr{L}(\mathscr{H})$ associated with the state $\operatorname{Tr} \rho(\cdot)$.

Next, we intend to construct a suitable conjugation $\mathscr{J}_k$ on $\mathscr{A}$. Let us observe that:

(i) The cone V_0 has the form

$$V_0 = \{A\rho^{1/2}; A \in \mathscr{L}(\mathscr{H}), A \geqslant 0\}^{\text{closure}}$$

(ii) The modular operator Δ for $(\mathfrak{M}, \rho^{1/2})$ is such that for some dense subset $\mathscr{A}^0 \subset \mathscr{A}$

$$\Delta\sigma = \rho\sigma\rho^{-1}$$

for $\sigma \in \mathscr{A}^0$. To show the latter, let C, B be in $\mathscr{L}(\mathscr{H})$. Then

$$\begin{aligned}((C\rho^{1/2}, \Delta B\rho^{1/2})) &= ((\Delta^{1/2}C\rho^{1/2}, \Delta^{1/2}B\rho^{1/2}))\\ &= ((\mathscr{J}\Delta^{1/2}B\rho^{1/2}, \mathscr{J}\Delta^{1/2}C\rho^{1/2}))\\ &= ((B^*\rho^{1/2}, C^*\rho^{1/2}))\end{aligned}$$

where $\mathcal{J}$ denotes the modular conjugation. Therefore, for any $C \in \mathcal{L}(\mathcal{H})$ and $B \in \{\bar{f} \otimes g; f, g \in D(\rho^{-1/2})\}$, where $(\bar{f} \otimes g)\, z = (f, z)\, g, f, g, z \in \mathcal{H}$, we have

$$\begin{aligned}\operatorname{Tr} \rho^{1/2} C^* \Delta B \rho^{1/2} &= \operatorname{Tr} \rho^{1/2} B C^* \rho^{1/2} \\ &= \operatorname{Tr} C^* \rho B = \operatorname{Tr} \rho^{1/2} C^* \rho B \rho^{-1/2} \\ &= \operatorname{Tr} \rho^{1/2} C^* \rho B \rho^{1/2} \rho^{-1}\end{aligned}$$

In particular, one has $\Delta \rho^{1/2} = \rho^{1/2}$ and $V_{1/2} = \{\rho^{1/2} A; A \in \mathcal{L}(\mathcal{H}), A \geqslant 0\}$.

Now, the following definition seems to be obvious.

Definition. 1. Let $\{x_i\}$ be the basis in $\mathcal{H}$, associated with ρ by its spectral representation. Then

$$Kf = K \sum_i (x_i, f)\, x_i \stackrel{\text{df}}{=} \sum_i \overline{(x_i, f)}\, x_i$$

(for $f \in \mathcal{H}$) is a well-defined conjugation on $\mathcal{H}$.

2. The map $\mathcal{J}_K(\sigma) = K \sigma K$ defines a conjugation on $\mathfrak{A}$.

The above defined $\mathcal{J}_K(\cdot)$ has the following properties: $\mathcal{J}_K(\cdot)$ maps V_0 into V_0 and $\mathcal{J}_K(\rho^{1/2}) = \rho^{1/2}$. As V_0 is a dual cone with respect to $V_{1/2}$, to prove the first property of $\mathcal{J}_K(\cdot)$ it is enough to show that the inequality

$$((\rho^{1/2} A', \mathcal{J}_K(A \rho^{1/2}))) \geqslant 0$$

holds for positive A, A' in $\mathcal{L}(\mathcal{H})$. Therefore, let us observe that

$$\begin{aligned}((\rho^{1/2} A', \mathcal{J}_k(A \rho^{1/2}))) &= \operatorname{Tr}(\rho^{1/2} A')^* \mathcal{J}_K(A \rho^{1/2}) \\ &= \operatorname{Tr} A' \rho^{1/2} K A \rho^{1/2} K = \operatorname{Tr} A' \rho^{1/2} K A K \rho^{1/2} \\ &= \operatorname{Tr}(A')^{1/2} \rho^{1/2} K A K \rho^{1/2} (A')^{1/2} \geqslant 0\end{aligned}$$

The second property of $\mathcal{J}_K$ is evident. Hence, one can conclude that $\mathcal{J}_K$ induces a reversing operation σ on $\mathfrak{M}$ (see Lemma 4.11 in ref. 8).

Next, let us take a self-adjoint operator $H^0 \in \mathcal{L}(\mathfrak{A})$ such that

(i) $[e^{iH_0 t}, \Delta^{is}] = 0, \qquad t, s \in \mathbb{R}$

(ii) $e^{iH_0 t} \mathcal{P} \subset \mathcal{P}$

(iii) $e^{iH_0 t} \rho^{1/2} = \rho^{1/2}$

where $\mathcal{P}$ denotes the natural cone. It is worth pointing out that such operators exist. Namely, let us consider a uniformly continuous, one-parameter group of *-automorphisms β_t on $\mathfrak{M}$ such that

$((\rho^{1/2}, \beta_t(A)\,\rho^{1/2})) = ((\rho^{1/2}, A\rho^{1/2}))$ for $A \in \mathfrak{M}$. Then, the properties (i)–(iii) follow from Corollaries 2.5.32 and 2.3.17 in ref. 2 and Lemma 2 in ref. 12.

Let us form $H = H_0 + \mathscr{J}_K H_0 \mathscr{J}_K$. Then

$$e^{iHt}\mathscr{P} \subset \mathscr{P}$$

and

$$e^{iHt}\rho^{1/2} = \rho^{1/2}$$

for $t \in \mathbb{R}$, where the first property of e^{iHt} follows from the Trotter product formula and the fact that $\mathscr{J}_K\mathscr{P} \subset \mathscr{P}$. Let us consider e^{iHt} for positive time, $t \geqslant 0$. It is easy to see (cf. ref. 2, Theorem 3.2.18; ref. 8, Theorem 4.12) that $\hat{\tau}_t = e^{iHt}$ induces a semigroup on $\mathfrak{M}$ such that detailed balance condition is satisfied for $(\mathfrak{M}, \tau_t, \rho^{1/2})$. Now we will consider perturbations of $\hat{\tau}_t$ (cf. ref. 2, Theorems 3.1.32, 3.1.33). As a first simple example we take the perturbation $P = -(\mathbb{1} - P_{\rho^{1/2}})$ where $P_{\rho^{1/2}}$ is the projection onto the subspace generated by $\rho^{1/2}$.

It is easy to check (again use the product formula) that the perturbed semigroup $\hat{\tau}_t^P$ is a $\mathscr{J}_K$-self-adjoint semigroup such that $\hat{\tau}_t^P\mathscr{P} \subset \mathscr{P}$. Moreover, $\hat{\tau}_t^P$ strongly commutes with Δ, $\hat{\tau}_t^P\rho^{1/2} = \rho^{1/2}$, and[3] $\lim_{t\to\infty}(\hat{\tau}_t^P)^*\,\hat{\tau}_t^P = \lim_{t\to\infty}\hat{\tau}_t^P(\hat{\tau}_t^P)^*$. Therefore, Conditions I and II are satisfied.

Next, we will describe a more complicated example. We take H as it was described before and we assume the dissipator D to be

$$D = (\log \Delta)^2$$

(dissipators λD, $\lambda \in \mathbb{R}^+$ can be treated in the same way),

Since

$$\exp[-t(\log \Delta)^2] = (2\pi)^{-1/2} \int d\mu(\lambda)\, \Delta^{i\lambda}$$

where $d\mu(\lambda) = \text{const} \cdot \exp(-\lambda^2/4t)\, d\lambda$ and $\Delta^{it}\mathscr{P} \subset \mathscr{P}$, we deduce that

$$e^{-t(\log \Delta)^2}\mathscr{P} \subset \mathscr{P}$$

Further, $e^{-t(\log \Delta)^2}$ is the semigroup of contractions and H is the self-adjoint operator in $\mathscr{L}(\mathfrak{A})$. Therefore, $e^{iHt - Dt}$ is the semigroup of contractions. Hence, the product formula implies

$$\hat{\tau}_t^D\mathscr{P} = e^{iHt - Dt}\mathscr{P} \subset \mathscr{P}$$

Having proved this, it is elementary that $(\hat{\tau}_t^D, \rho^{1/2})$ satisfies the detailed balance condition.

On the other hand, e^{iHt}, $t \in \mathbb{R}$, commutes strongly with Δ. Thus, $\hat{\tau}_t^D$ is

[3] Since $[iH + P, -iH + P]\, f = 0$ for an arbitrary vector $f \in \mathfrak{A}$.

the normal semigroup and condition II is satisfied. So we can conclude that $\hat{\tau}_t^{\lambda D} = e^{iHt - \lambda Dt}$, $t \in \mathbb{R}^+$, provides a class of dynamical semigroups for which Conditions I and II are satisfied.

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